Short to medium term morphological prediction of river erosion and planform change: advance knowledge for decision support system in river monitoring and erosion management

Kumar Nikhil[1*], Hassan Kazi[1], Kumar Mani[1] and Kumar Anil[1,2]

1 Mathematical Modelling Centre, FMISC, WRD, PATNA, India
2 Flood Management Information Support Centre (FMISC), PATNA, India
*Corresponding author, e-mail: enikhilkumar@gmail.com

Abstract

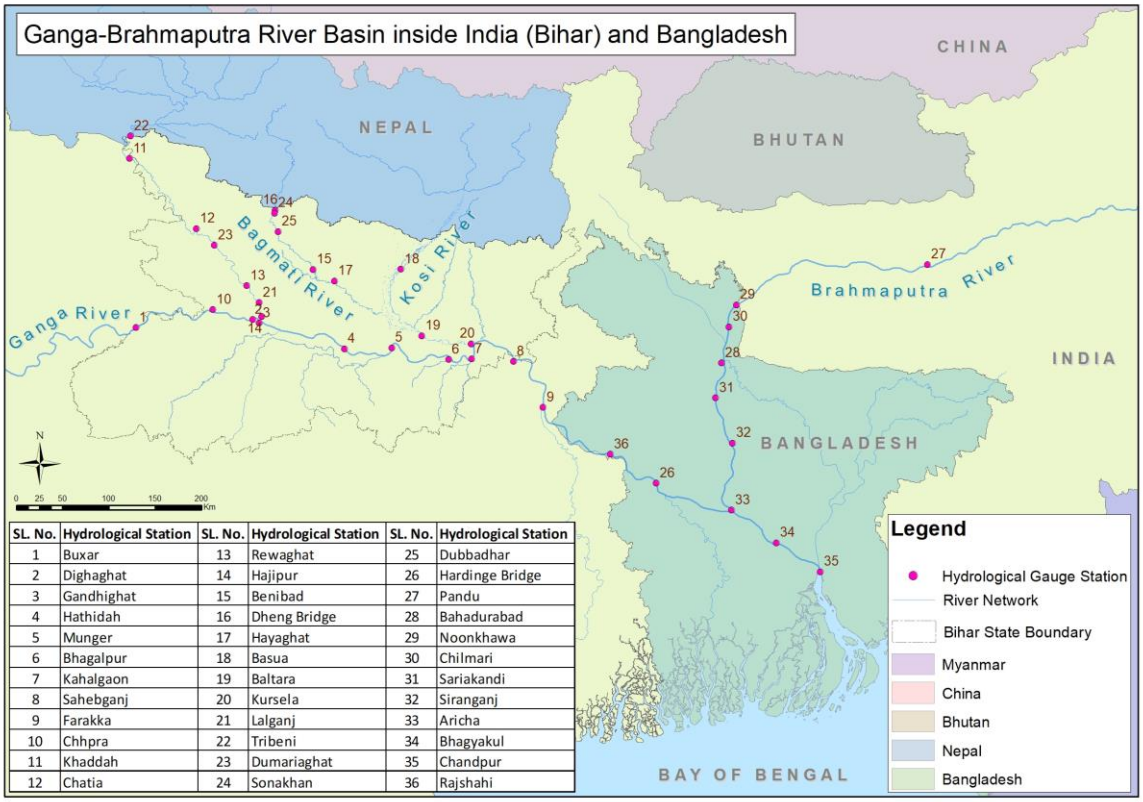
Extreme variations in monsoon flow and excessive transport of fine sediment in alluvial rivers in India make them morphologically very dynamic. Development of sand bars, bend scour, general river bed erosion and deposition, bankline shifting, channel abandonment, outflanking and neck cut-off/loop cut development are recurrent phenomenon in meandering and braided rivers. They create additional erosion and flooding risks. Two-dimensional sediment transport and morphological modelling in curvilinear body fitted computational grid is a useful tool for making short to long-term prediction (Enggrob and Tjerry, 1999; Hassan et al., 2002). The model results are able to identify future vulnerability of erosion and deposition, bankline shifting, and thus, aggrading and degrading river reaches (Figure 1). In the highly eroded river bend, particularly at oxbow, there is potential risk of future loop-cut development. At aggrading reaches, flood water level can rise considerably for same condition of flow of the past (Maulishri et al. 2019), and thus, can cause flooding by compromising the embankment height; some damage to the flood embankment are inevitable in such adverse development. Bank erosion can also increase in such aggrading reach as river will need conveyance to pass the flow. The present 2-D model has predicted morphological development for three monsoon periods (2019, 2020 and 2021 (Figure 1), and has identified several locations (1 to 5) of vulnerabilities on bend scour, bar development, bank erosion, river aggradation and increased risk of flooding, and have recommended immediate monitoring work to be commissioned in those reaches, some of the reaches require implementation of erosion protection works. Long-term morphological prediction for 20 to 50 years can guide establishing long-term river monitoring programme and masterplan for river behaviour management.

Keywords: erosion prediction; MIKE21C; bank erosion; 2-D modelling

1. Introduction

Bagmati is the major tributary of the Kosi river in Bihar (Fig 1). Kosi then flows to the Ganga River. This tributary river has flashy discharge (Ghimire et at 2013). The sediment is transported during high water levels and deposited in low water levels. Increase in bed levels, shift in channel course, split off the bank and bank erosions are very common during heavy rainstorm. Abrupt reductions in channel gradient, common in alluvial fans due to presence of pools and riffles, are abundant, and may trigger unexpected bed aggradations raising the channel bed above the surrounding terrain. This may cause an avulsion which sends the channel to another part of the fan (Matsuda, 2004; Legg and Olson, 2014). Reworking of sediment deposits by erosion, transportation and deposition downstream is a common phenomenon. Flood embankments constructed on a channel running on such an unstable zone, the river bed poses risk of become higher than the surrounding land surface, which could cause channel avulsion (Matsuda, 2004). Since the river is unstable, the morphology is dynamic and because the alluvial fans formed have no well-defined limits; paleo and recently abandoned channels, oxbow lakes are common characteristic features. The sediments in the catchments are delivered from the Lesser Himalaya and the Siwaliks.

Ganga and Brahmaputra river system carries the highest sediment load (Knighton, 1998) in the world. Bagmati, ultimate flowing to the Ganga through Kosi river, has also very high sediment transport rate. Average annual sediment load at Dheng Bridge (India-Nepal border) is 10 Mt/year (Sinha et al. 2019). Discharge varies widely between monsoon and dry season; average annual discharge is 156 m^3/s (Sinha et al. 2019); flow at start of monsoon could be as low as 40 m^3/s, and dry season unrecorded flow is far lower than this. Design discharge for flood embankment is 8,245.7 m^3/s (CWPRS, 2010-11). Such extreme variations in monsoon flow and excessive transport of fine sediment make causes considerable morphological changes to the river planform, bed aggradation and degradation and bank erosion.



SL. No.	Hydrological Station	SL. No.	Hydrological Station	SL. No.	Hydrological Station
1	Buxar	13	Rewaghat	25	Dubbadhar
2	Dighaghat	14	Hajipur	26	Hardinge Bridge
3	Gandhighat	15	Benibad	27	Pandu
4	Hathidah	16	Dheng Bridge	28	Bahadurabad
5	Munger	17	Hayaghat	29	Noonkhawa
6	Bhagalpur	18	Basua	30	Chilmari
7	Kahalgaon	19	Baltara	31	Sariakandi
8	Sahebganj	20	Kursela	32	Siranganj
9	Farakka	21	Lalganj	33	Aricha
10	Chhpra	22	Tribeni	34	Bhagyakul
11	Khaddah	23	Dumariaghat	35	Chandpur
12	Chatia	24	Sonakhan	36	Rajshahi

Fig 1: Location of Bagmati River in Bihar, Kosi and Ganga Rivers and other major tributaries in Bihar

The dynamic river behaviour causes damage to agricultural land, road networks, settlements, river training works and other infrastructures such as bridges and culverts. It becomes extremely difficult and expensive as well for planners and designers to carry out monitoring, preparedness and response activities in long reaches of rivers. In most cases, response and recovery works are mostly carried out only at affected reaches after the damages were done. Similar to flood forecasting, river erosion prediction can help minimising damage and can help planners and designers, in advance, to effectively formulate (within reasonable budget) their monitoring and preparedness works and design of structural response, both for flood control and erosion control. There are several types of structures designed according to their function. Structure for flood control is sometimes referred as structures for 'high water training' or 'training for discharge'. It aims at the provision of a sufficient cross-sectional area for the safe passage of the maximum flood without an attempt at changing the river bed conditions. Structures for erosion and sediment control are referred as structures for 'mean water training' or 'training for sediment'. This type of training aims at rectification of river bed configuration and efficient movement of suspended and bed load for keeping the channel in good shape. The maximum eroding capacity of river occurs in the vicinity of mean water or dominant/bankful flood discharge.

In the present paper, medium-term (3- years) morphological prediction has been issued for Bagmati river, in one of its most active stretch from Dheng Bridge (Indo-Nepal border) to Benibad, using a two-dimensional sediment transport and morphological model. A typical discharge hydrograph of one monsoon has been repeated for three monsoons to generate characteristic flows through the river. The hydrograph represents high flows as well as bankful and average annual discharge, and thus, forms an ideal hydrological forcing for simulating morphological changes over a period of three years.

2. Two-dimensional (2-D) morphological model of Bagmati

2.1 Introduction

A two-dimensional (2-D) unsteady sediment transport model of the Bagmati from Dheng Bridge to Benibad gauging station has been developed. The model is 105 km long; capable of predicting river bed and bank erosion. Thus, the model could be applied for prediction of future morphological development and can be used for formulating monitoring and response programme, and design of erosion control structure (Kumar et al. 2019). The model, developed in MIKE21C (DHI, 2016), simulates 2-D Saint Venant equations of conservation of mass, and momentum. The equations are transformed into curvilinear co-ordinates (DHI, 2016).

2.2 MIKE21C modelling software

MIKE 21C is a generalised mathematical modelling system for the simulation of hydrodynamics of vertically homogeneous flows, and for the simulation of sediment transports. The technology is among the best 2-D morphological modelling software (Langendoen, 2001). Key elements in MIKE21C modelling tool are:

- Curvilinear grid (movable adaptive grid to account bank erosion, sandbar erosion)
- Hydrodynamics
- Helical flow-analytical model
- Bed and suspended load
- Alluvial resistance
- River bed morphology
- Bank erosion
- River planform

Based on the calculated bed and suspended load, the erosion and deposition of river bed are found from solving the sediment continuity in Eq. (1) and bank erosion rate described by the empirical relation in Eq. (2).

$$(1-n)\cdot\frac{\partial z}{\partial t}+\frac{\partial S_x}{\partial x}+\frac{\partial S_y}{\partial y}=\Delta S_e \qquad (1)$$

$$E_b=-\alpha_b\cdot\frac{\partial z}{\partial t}+\beta_b\cdot\frac{S}{h}+\gamma_b \qquad (2)$$

Where α_b, β_b and γ_b are calibration co-efficient (DHI, 2016). The position of the land-water border of the modelling area is recalculated based on the simulated erosion $E_b.\Delta t$ and the computational grid is updated during simulation. The eroded bank material is included in the sediment continuity equation.

The 3-D secondary (helical) flow created due to curving streamlines in a river bend or around an island or mid-channel bar causes a small deviation δs in the direction of flow velocity near the bed and therefore also the bed shear stress, as schematically shown in Fig 2.

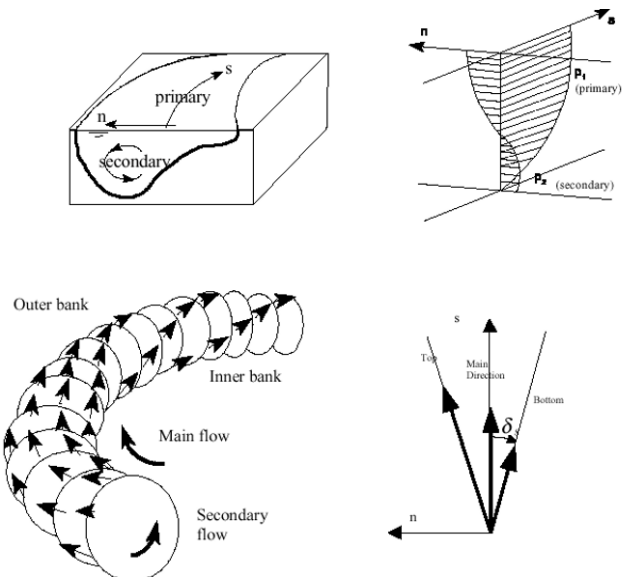


Fig 2: Theoretical illustration of secondary flow in river bend (Source: MIKE21C Reference Manual, DHI, 2016)

The direction of the bed shear stress in a curved flow plays a crucial role in a bed topography model for river bends. The bed shear stress direction is specified as (Rozowskii, 1957, Engelund, 1974, Struiksma et. al, 1985):

$$\tan \partial_s \approx -\beta \frac{h}{R_s} \qquad (3)$$

β is defined as:

$$\beta = \alpha \frac{2}{\kappa^2} (1 - \frac{\sqrt{g}}{\kappa C} \qquad (4)$$

The approximate value of β is 10. In regions of changing curvature of the streamlines, the secondary flow will adapt gradually. The adaptation of the secondary flow profile is considerably faster near the bottom (where the bed shear stresses act) than further up in the water column.

Bed and suspended load is calculated on the basis of empirical sediment transport formulae (Engelund-Hansen, 1967; Ackers and White, 1973; Van Rijn, 1984). In contrast to suspended load, it is assumed for the bed load that it responses immediately to changes in local hydraulic conditions without any time and space lag. A more generalized form of Engelund-Hansen formula can be expressed as:

$$S \approx \frac{0.01 \cdot \sqrt{C} \cdot I\sqrt{I}}{\sqrt{w \cdot d}} \cdot Q\sqrt{Q} \qquad (5)$$

Where S is total load, C is Chezy's friction coefficient, I is hydraulic gradient, Q is total discharge, w is river width and d is grain size

2.3 Bagmati 2-D model

The 2-D model is 105 km long from Dheng Bridge upto Benibad gauging station. The 2-D model's bathymetry was prepared using surveyed cross-sections of pre-monsoon 2014. The 105 km long model is built on 143,883 computational cells in curvilinear grid. The grid was generated in multi–block grid generation technique to maintain grid orthogonality within the allowed tolerance specified in MIKE 21C. Numerous blocks of sub-grid were generated first and then merged together to form the final grid. The grid orthogonality was maintained within MIKE21C recommended tolerance limit between –0.05 and +0.05 for over 90% of the model domain and the grid aspect ratio is considered satisfactory as the ratio falls between 2 to 8 (MIKE21C recommended value is 2 to 10) for almost 100% area of the model domain. Building grids in blocks gave control in resolving distribution of flow across channel width, along bends, and flow paths along the meandering mainstream. Such control on grid resolution helps better computation of general erosion/deposition, bend scour, obstruction scour, and bank erosion. There are 1,971 computational cells along 105 km length of the river, and 71 cells across the width of the river. This provides average cell size as less than 40m along the river length and as 2 to 10 m across the width of the river. Such grid resolution is generally sufficient for resolving localised phemenon, e.g., bend scour, bar development, secondary currents (Hassan et al, 2002).

Discharges and water levels were used as hydrological boundary condition to the 2-D model. Bagmati flow at Dheng Bridge and a right bank tributary flow, called Lalbakiya, were used as inflows; water level at downstream boundary was used as outflow condition. Chezy's friction coefficient C was used as roughness; spatially varying roughness field was used; shallower areas have higher roughness coefficient than the deeper channel areas. Minimum and maximum C values were 25 and 50 in the model. Fine uniform sand of 0.15mm was considered as median (D_{50}) grain size. Van Rijn (1984) bed and suspended load formula was used for sediment load calculation. Equilibrium bed level was considered at upstream and downstream boundary condition for the sediment transport and morphological simulation.

2.4 Model calibration

The model has been calibrated for 2017 monsoon flow against observed water levels (Table 1 for 2017) and water surface slopes. The model calibrates well against observed water level peak, against water surface slope as shown in Fig 3. The model also calibrates well against historic water surface slope between the two gauging stations, one at Sonakhan and the other at Dubbadar. Historic observed average monsoon slope from 2013 to 2017 was found as 0.30m/km, and the model simulated slope was found as 0.28 m/km, and thus, the model is considered well calibrated and should have good predictive ability for future morphological changes.

River name	Gauge station name	River chainage (m)	Observed water level (m)	Modelled water level (m)	Difference (m)
Bagmati	Sonakhan	4190	70.70	70.54	0.16
	Dubbadhar	33179	63.62	63.99	0.37
	Kansar	49116	60.10	60.60	0.50

Table 1. Model calibrated water levels at three key gauge locations along the Bagmati River

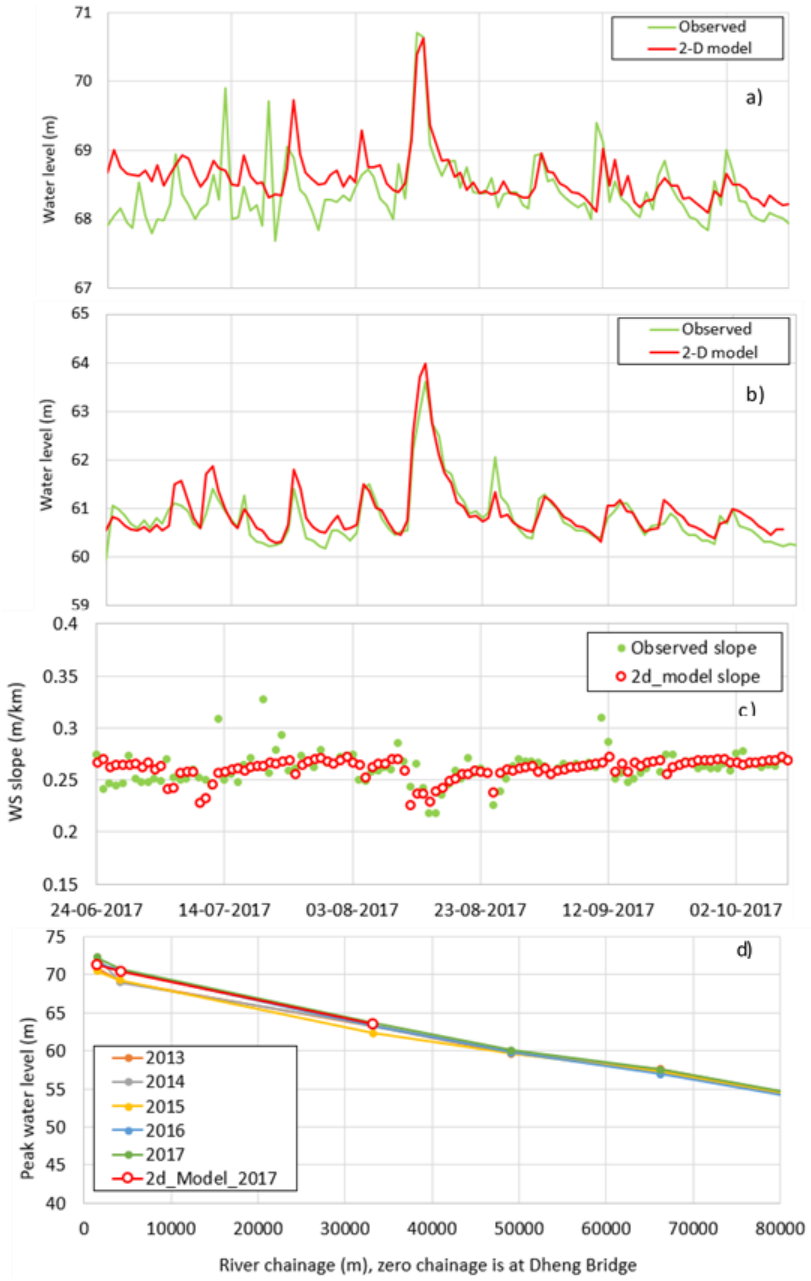


Fig 3: Hydrodynamic calibration of 2-D model: water level at Sonakhan gauging station (a), Dubbadhar gauging station (b), water surface slope between Sonakhan and Dubbadhar (c) and historic peak water level profile (d)

Model predicted key results are presented in Fig 4 for peak flow condition of 2017 (1,291 m^3/s). Distribution of water depth (a), speed contours (b), sediment concentration (c), and shear stress (d) demonstrate key vulnerable reaches. Using distribution map of these parameters, better informed decisions are possible to make for planning new erosion management works, monitoring of scour and bathymetry. In general, all reaches and bends of red colour contours for speed, sediment concentration and shear stress require more attention; using detail parameter values (available in result files), the critically vulnerable reaches could be identified and new erosion management and monitoring works could be taken up. Medium-term morphological prediction has been made in a sub-scale model for the first 35km from the upstream end (Dheng Bridge) and discussed next.

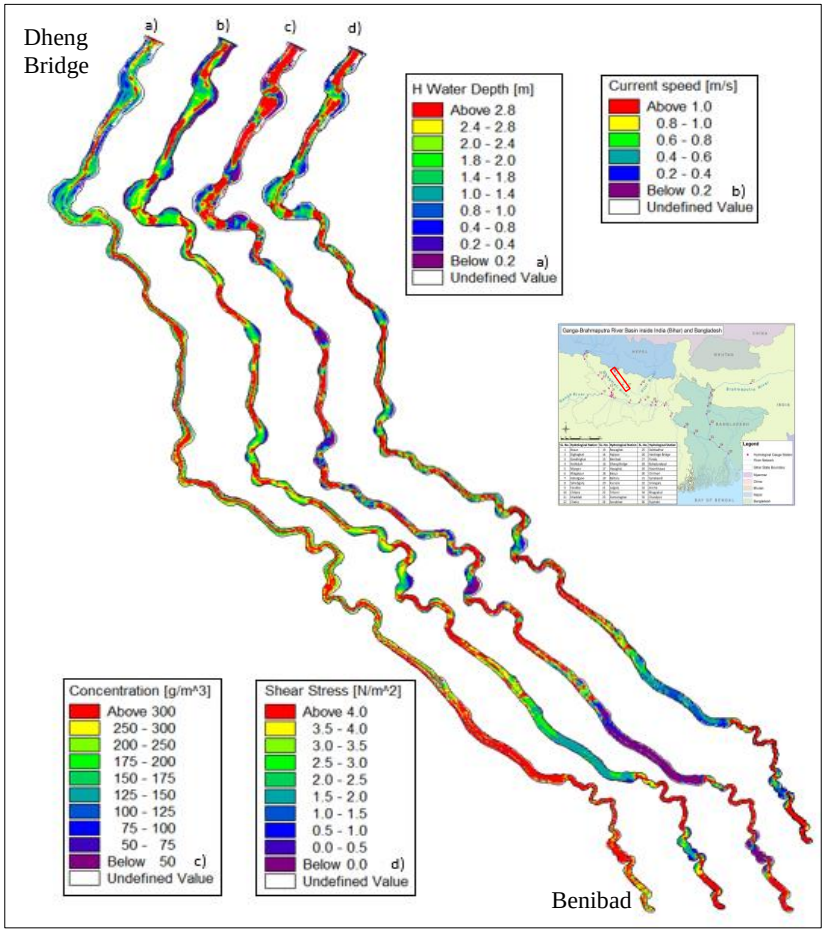


Fig 4: Model result-distribution of depth (a), speed contours (b), sediment concentration (c), shear stress (d); location of 2-D model shown inset in context to Bihar, Nepal and Bangladesh

3. Model application for medium-term morphological prediction

3.1 Introduction

Medium-term model prediction has been carried out for three monsoons, for 2019 to 2021; the monsoon (15^{th} June to 15^{th} October) of 2017 has been repeated three times to use as upstream inflow condition to the 2-D model. The three monsoons' hydrograph is shown in Fig 5 (bottom frame); all flow below 200 m^3/s has not been considered in morphological simulation to avoid modeling instability in very shallow condition where the 2-D model accounts for flooding and drying in the calculation. The morphological developments have been evaluated at specific time on the temporal scale over the three-monsoon period; the specific temporal points are shown with red dots in Fig 5.

3.2 Bed aggradation, degradation and erosion vulnerability

Medium-term morphological prediction over three-monsoon has identified vulnerable areas of bend scour, mid-channel bar growth and flow bifurcations, oxbow meander, aggrading and degrading

reaches. Several vulnerable points/reaches have been identified as indicated by Arabic numbers, 1 to 5, in Fig 5, in the far right alignment of the channel. In some of the reaches, there is almost no margin with flood embankment from the eroded banks, and thus, a monitoring programme should be commissioned by the next monsoon; even erosion arresting measures may be required, e.g., around point 1 on left bank and point 3 on right bank; at both places flood embankment has almost zero margin with the souring river bend.

Bend scour and bed erosion will continue to develop and will increase in 2nd and 3rd monsoon. Bend scour develops at outer bend at left and right bank around point 1, 2, 3 and 4. Bend scour at point 2 has led to the development of a sand bar at immediate downstream attached to the right bank (the red contours between point 2 and 3 attached to the right bank); this sand bar grew over the first monsoon, and then started moving downstream and disappearing in the 2nd monsoon. This bar (attached to bank) disappeared by third monsoon, and this led to the development of the mid-channel sand bar between point 3 and 4, just at immediate upstream of the oxbow. The development and erosion of the attached sand bar to the right bank and the mid-channel bar are distinctly dynamically linked. As the sand bar (attached to bank) grew, moved and eroded, the mid channel bar, at immediate downstream, gradually developed. The mid-channel bar would pose number of vulnerabilities; it will increase bend scour and bank erosion at the outer bend along the right bank and also at the outer bend at the oxbow along the left bank.

The morphological development as described above will lead to aggrading reach at downstream of the oxbow bend. Aggradation develops along the mid-channel, generally above 1.5m in magnitude; the length of the aggrading reach is over 10km; this is expected to create flood vulnerability on both banks (see next section).

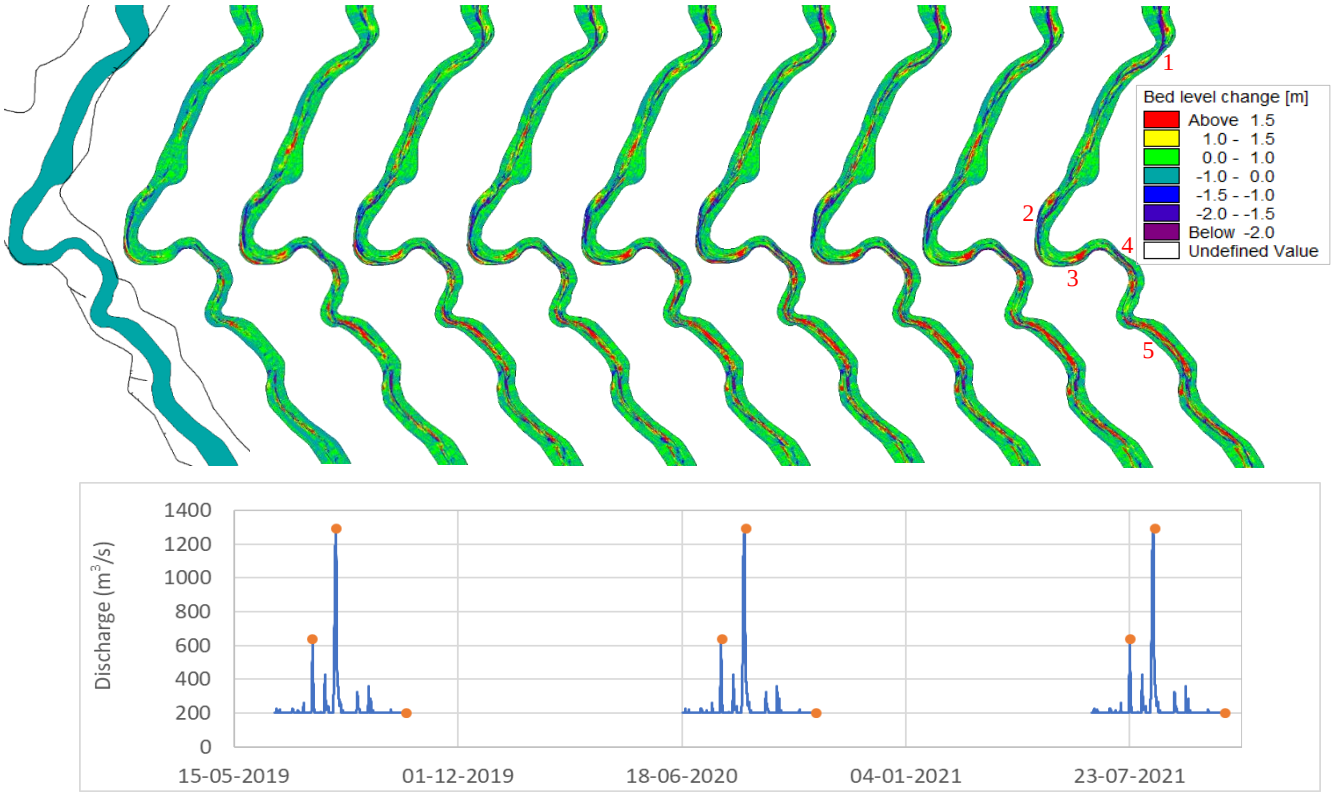


Fig 5. Shor-term prediction of erosion vulnerability for three monsoons; erosion-deposition map (no erosion or deposition at start of simulation, and nine maps, sequentially shown at red-dots of the hydrographs (order from left to right)

While the trend of development of erosion and deposition at specific time are important to know as demonstrated in Fig 5, e.g. at peak of the monsoon when higher erosion expected due to higher discharge and at end of monsoon at low flow when deposition is expected, it is also very vital to quantify the absolute maximum value of erosion (and deposition too). Such absolute maximum erosion can occur, even at relatively low flow than the highest peak at critically higher hydraulic gradient (Eq.5). Absolute maximum magnitude of erosion and deposition, predicted over the three monsoons, is depicted in Fig 6. Over 5 m of bed erosion has been predicted along the outer bends. Over 1 m of deposition has been predicted, particularly along the main course of the channel in the downstream reach. Such high bed erosion (over 5m) combined with local scour will trigger bank erosion and will also damage existing protection works (e.g., spurs or revetment works). Referring to the numeric location identifier in Fig 5, it is evident that around Point 1 at left bank, and Point 2 at right bank, the bend will experience continued erosion; no deposition is detected there. On the other hand, at Point 3 and Point 4, though there is also high erosion, some extent of deposition will also occur along the bank during low flows. And in the downstream reach (Point 5 and further down), there are combination of erosion and deposition, though deposition is considered domination processes there as mentioned earlier.

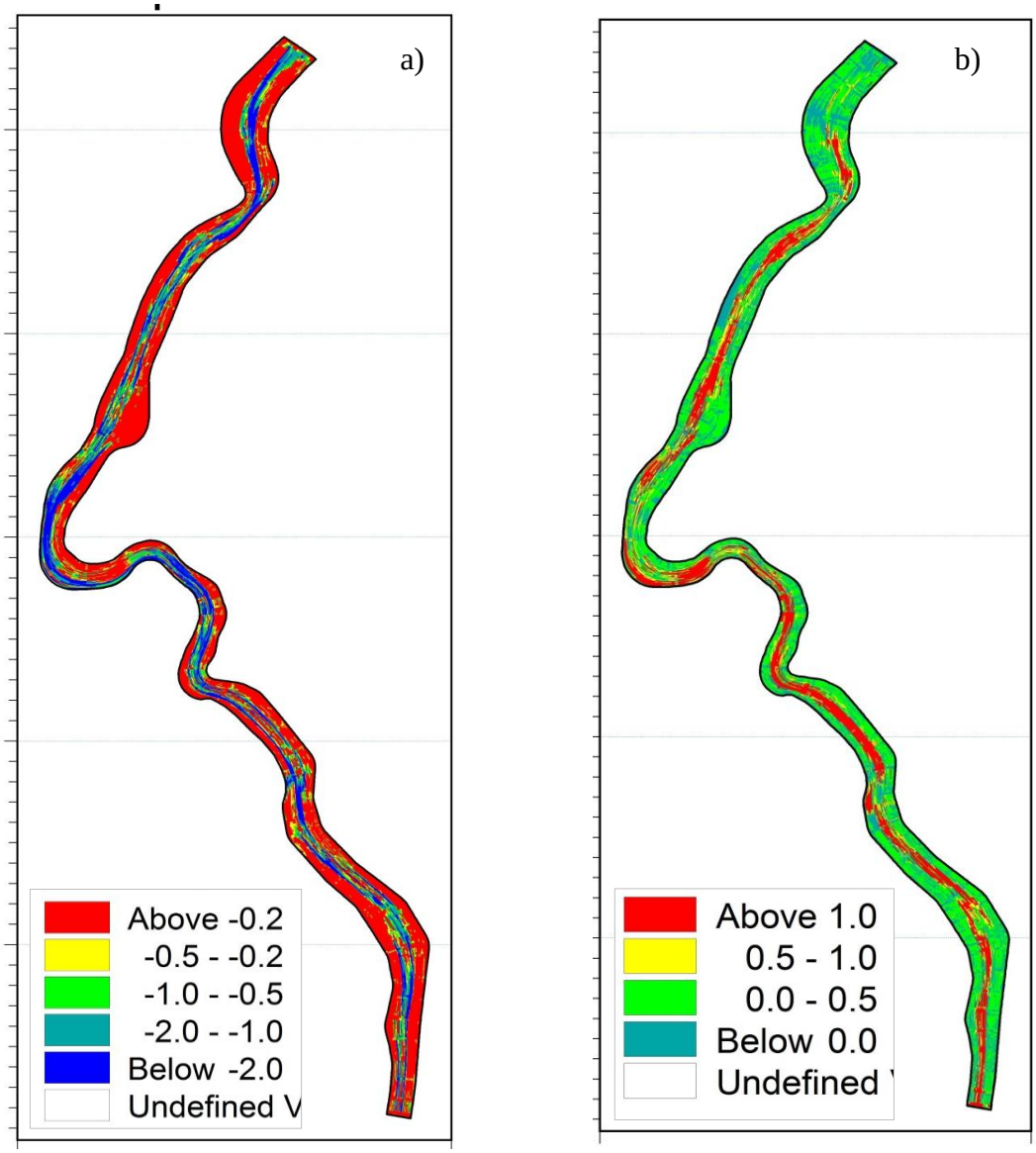


Fig 6. Absolute maximum magnitude of degradation (a) and aggradation (b) predicted over three years of monsoon period

3.3 Bank erosion vulnerability

Bank erosion, along both banks, have increased over the years (Fig 7), particularly in 2nd and 3rd monsoon, due to the growth of bend scour and multiple sand bar development in the reach between points 2, 3 and 4. Along right and left bank, the rate of bank erosion is higher in the 2nd and 3rd monsoon than in the 1st monsoon. Along the right bank, between chainage 15,000m and 20,000m, maximum accumulated bank erosion over 1st monsoon is approximately 10m, while by the end of 3rd monsoon, the accumulated bank erosion is over 35m. Similarly, at the same location at left bank, there was very minor bank erosion by the end of first monsoon, but has increased to higher rate by the end of third monsoon, although the erosion is still considered minimal. However, the concern is, if the bank erosion continues on both left and right bank at this oxbow, this will create risk of development of loop cut at this bend, which will create more flood and erosion vulnerability at immediate downstream.

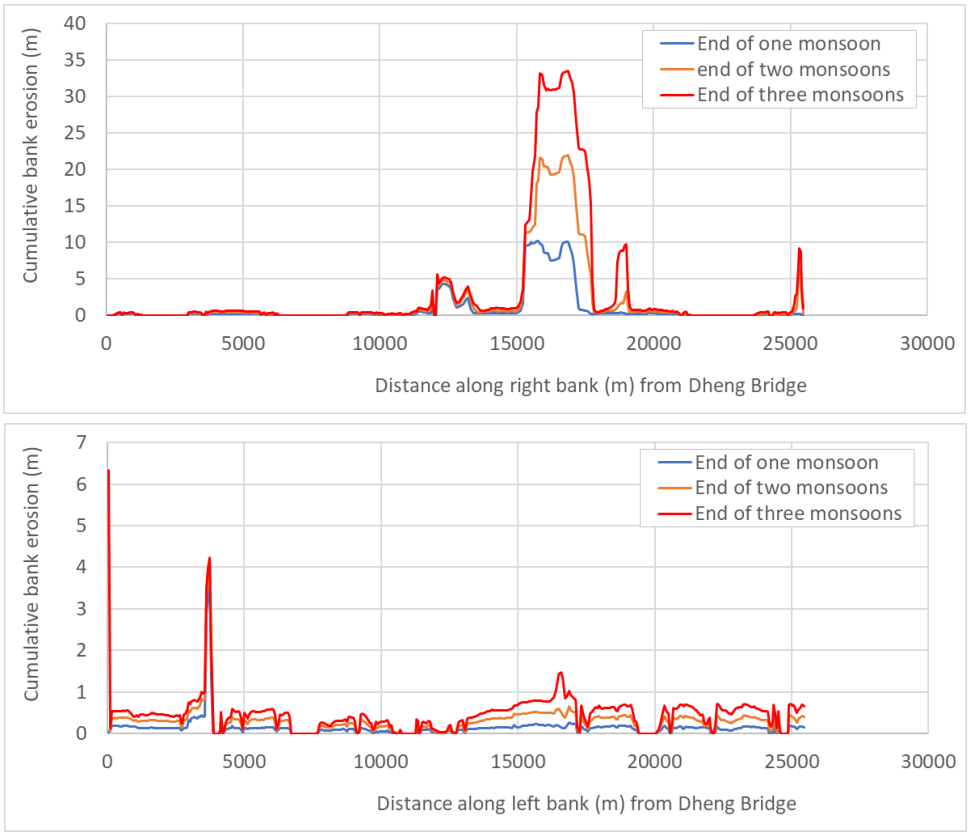


Fig 7. Shor-term prediction of bank erosion along right bank (a) and along left bank (b)

3.4 Flooding vulnerability

Morphological developments change flow paths, modify areas and flow conveyances, and thus, influences flood levels (Hassan et al. 2002). Net aggradation in river bed will induce rise in water level, while net degradation will induce fall in water level for same flow condition (Fig 8). There is significant fall in water level in the upstream reach of the river due to net degradation effect. Similarly, in the downstream reach rise of water level over a long reach of river aggradation. The rise and fall of water level in the degrading and aggrading reaches have been quantified in Figure 8. In the degrading upper reach, the river level can fall by over 03.m over 10km, while in the lower aggrading reach, the river level can rise by about 0.2m. The rise in the river level is of particular concern for flooding; 0.2m rise is a considerable increase; even if the flood embankment crest may not compromise due to this rise, there is need for increased monitoring in this reach, as rise in flood level is, in general, creates higher risk for increased breaching possibility.

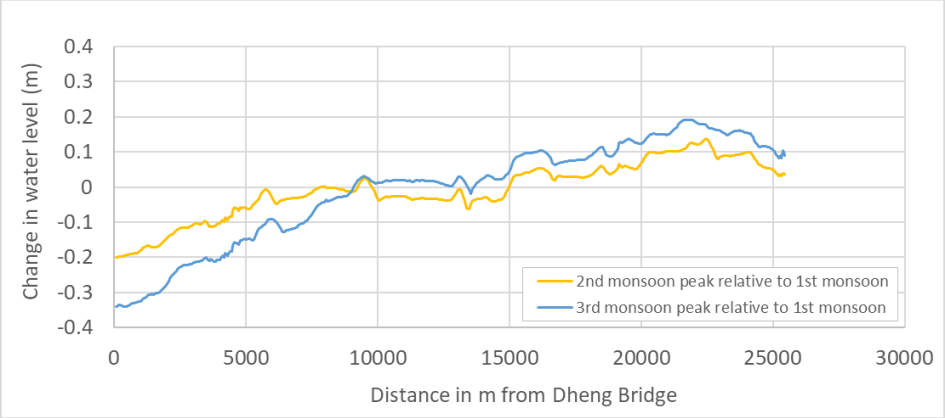


Fig 8. Shor-term prediction of flood vulnerability for three monsoons: change in water level at peak flow condition in monsoons 2 and 3 relative to monsoon 1 (negative values are fall and positive values are rise in water level).

4. Conclusions

Two-dimensional sediment transport and morphological model in curvilinear body fitted computational grid is a useful tool for making short to long-term prediction. A 2-D model of the Bagmati River, which is morphologically very active, has been developed in MIKE21C modelling software, one among the best morphological modelling technologies and presented in this paper. The 2-D model generates 3-D effect through inclusion of secondary current, which is vital at river bend, where river bank is critically affected from bank erosion. The model is well calibrated against observed water levels of 2019 monsoon, and also calibrates well against the historic hydraulic gradient from 2013 to 2017. Thus, the model is able to predict future vulnerability of erosion and deposition, bankline shifting, and thus, aggrading and degrading river reach.

The present 2-D model has predicted morphological development for three monsoon periods (2019, 2020 and 2021), and has identified several locations of vulnerability. Over 5 m of absolute magnitude erosion has been predicted in few bends; combined with local scour, such high erosion will trigger bank erosion and will also damage existing erosion protection works. In the highly eroded river bend, particularly at oxbow, the potential of future loop-cut can be detected. At aggrading reach, flood water level can rise considerably for same condition of flow of the past, and thus, can cause flooding by compromising the embankment height; some damage to the flood embankment are inevitable in such adverse development; bank erosion can also increase in such aggrading reach as river will need conveyance to pass the flow.

Immediate monitoring work should have to be commissioned in those reaches; some reaches may require immediate implementation of erosion protection work. Long-term prediction can guide establishing long-term river management and monitoring plan; future long-term prediction, say for 20 to 50 years, is also recommended.

Acknowledgments

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