Estimating field-scale variability in soil saturated hydraulic conductivity from rainfall-runoff experiments

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OVERVIEW

- Introduction and Background
- Research question
- Objectives of the study
- Experimental system
- Results
- Conclusions
- Limitations

Saturated soil’s ability to transmit water when subjected to hydraulic gradient

- Influences partitioning of rainfall into surface and subsurface waters
- Exhibits maximum variability among infiltration parameters (Russo and Bresler, 1981).

We deal with field-scale estimation

- Typically $K_s$ is assumed to be spatially log-normal

$$f_{K_s}(k) = \frac{1}{\sqrt{2\pi\sigma_y^2 k}} \exp\left[-\frac{1}{2} \left(\frac{\ln k - \mu_y}{\sigma_y^2}\right)^2\right]$$

If $r < K_s$ and $t_r < t_p$, then $f = r$ and surface saturation does not occur, and no surface runoff is generated!
For a fixed rainfall rate, $r$, at any time, the maximum rate of infiltration can only be $r$.

Experimentally obtained “effective” $K_s$ depends on rainfall (Langhans et al., 2011; Ojha et al., 2017)
Since each field-scale rainfall-runoff experiment resolves only a part of the variability in $K_s$, the following issues arise:

- **Identifiability**: part of the $K_s$ space is not resolved
- **Non-uniqueness**: each experiment yields a different estimate of $K_s$ distributions
- **Generalization**: the results can not be generalized even for a study area
- **Equifinality**: model and measurement errors also lead to non-uniqueness
1. Use a field-averaged infiltration* model and Monte Carlo simulations to obtain the possible range of distributions of $K_s$ that would describe experimental observations over a field for a rainfall event.

2. Consolidate the ranges of $K_s$ distributions over multiple rainfall events to yield the best range of $K_s$ distributions, using an information-theoretic approach.

EXPERIMENTAL SYSTEM

- Closed plot system with silty loam soil
- Experiments conducted under natural rainfall events over a period of a year


Adapted from Flammini et al. (2018)
Rainfall rate and infiltration rate observed during the November 02, 2013 event in the study plot
**METHODOLOGY**

\[ y(t) = P(x, t; \theta) + \varepsilon \]

- **Model inputs:** \( r, \Delta \theta, \) and \( \psi \)
- **Model parameters:** \( \mu_y \) and \( \sigma_y \)
- **Error model:** Gaussian

**Model inputs:**
- \( r \), \( \Delta \theta \), and \( \psi \)

**Model parameters:**
- \( \mu_y \) and \( \sigma_y \)

**Error model:** Gaussian

- **Expected value of infiltration rate**
- **Infiltration model** (Govindaraju et al., 2001)
- **Model inputs:**
- **Model parameters:**
- **Measurement error**
Stable distribution achieved at $t = 3.5$ hours
Stable distribution achieved at $t = 1.5$ hours
Stable distribution achieved at $t = 3.5$ hours
• Entropy-based measure to consolidate the stable distributions

Summary distribution of the 8 calibration experiments
COMBINING THE DISTRIBUTIONS

Marginal distributions of the parameters $\mu_y$ and $\sigma_y$
## PARAMETER ESTIMATES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Median</th>
<th>Most likely</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{K_s}$(mm/h)</td>
<td>0.87</td>
<td>16.32</td>
<td>2.98</td>
<td>1.27</td>
</tr>
<tr>
<td>$\sigma_{K_s}$(mm/h)</td>
<td>0.01</td>
<td>1041.20</td>
<td>8.93</td>
<td>0.86</td>
</tr>
<tr>
<td>$cv_{K_s}$</td>
<td>0.01</td>
<td>63.80</td>
<td>3.02</td>
<td>0.68</td>
</tr>
</tbody>
</table>

**Experimental values:**
- $\mu_{K_s} = 1.0$ mm/h
- $cv_{K_s} = 1.0$
Eight separate rainfall-runoff experiments used

Comparison of observed field-scale infiltration rate and simulated infiltration rate using the most likely parameters

$R^2 = 0.984$

$NS = 0.980$
LIMITATIONS

- Monte Carlo analysis used in the study
- The support in the parameter space for each rainfall-runoff experiment may not be similar
- The summary measure requires common support
- What if point measurements of $K_s$ are available independently in the field from infiltrometers? No good way to leverage these data.
CONCLUSIONS

- Rainfall-runoff experiments may not explore the entire log-normal space of saturated hydraulic conductivity.

- Calibration using different rainfall-runoff events may not result in the same set of $K_s$ distributions. The resulting distributions may also have different forms.

- The summary measure can result in a more principled estimate of $K_s$ for a given study area.
Thank you!