Modeling of Lumber Vertebrae of Spine Using Bondgraphs

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Abstract

The vibration of human body especially vertical column is considered as one of the major risks associated with the back disorders. These disorders are mainly due to the various dynamic stresses developed by the compression of the spine. The accurate modeling of the lumber vertebrae of spine is one of challenges for the modellers to obtain such stresses and forces. The main focus of this paper is to model the lumber vertebrae through bond graph modeling taking into consideration its physical correspondence with the system. In this work, a bond graph model of seven rigid elements for lumber vertebrae has been modelled. Rigid elements of vertical column have been modelled as Rayleigh beam elements, which account for bending as well as rotary inertia. The muscle strength and intervertibral fluid has been taken as spring-damper system. The flexural rigidity of the model also depends on the compression of lumber vertebrae. In this work, T₁₂-L₁- L₂- L₃- L₄- L₅- S₁, these seven elements have been modelled sequentially. The overall nonlinear stiffness of the various segments is also considered in the model. Besides this, different postures of the spine with excitation base have been considered for modelling. The strength of the muscle and intervertebral fluid has been modelled considering six spring mass damper system between the rigid elements.

Key words: Bond graph modeling, Lumber vertebrae, Flexural rigidity, Dynamic stresses

1. Introduction

Low back pain is one of the common reasons for days away from work according to the Bureau of Labour Statistics data. This disorder accounts for major economic loss in terms of direct and indirect costs associated. This occupational low back pain represents a significant public health problem and causes an economic burden to employers. To overcome the lower back related problems and maximizing profit, are the powerful drives for research in this field. Many industries and organisations are investing for research in this sector. While going through the literature, a lot of studies are carried by various researchers in this field. Recently, (Hauges and Nelson, 2008) have discussed the economic loss caused by the low-back pain to Industries. (Ayari et.al, 2009) evaluated the injury risk to vertebrae, when it exposed to vertical vibration. They have presented a finite element model of the vertebrae in order to compute the model parameters, dynamic stresses and other forces. (Bazrgari et.al, 2008) have computed the trunk muscle forces and other parameters besides incorporating the stability issue. In these lines, (Khalid and Daniel, 2008) compared the rest allowances model for ergonomic point of view. This study is mainly limited to static muscular work. Many other studies are there, which have been modelled by either Finite Element or the other; but the main purpose of this paper is to model the lumber vertebrae through bond graphs (Mukherjee et.al, 2007) and validate the model with other available studies.
These occupation stresses at the low back region are caused by dynamic stresses, which are mainly due to the compression of the spine producing micro-fracture in the endplates and the lumber vertebrae. The long-term exposure of the human body to vibration may lead to mechanical fatigue and micro-fracture in the vertebrae’s and micro lesions in the intervertebral disc. The present research is focussed to study the dynamic behaviour of the vertebrae of the spine. Only lumber vertebrae’s are considered in the present study, which incorporates seven rigid elements including the endplates. The muscle strength and intervertebral fluid has also been considered in the model and henceforth; biomechanical modelling is done using bond graphs, as it is essential to calculate the various stresses and deflection at specific points in various spine regions. Bond graphs may be conveniently used as a modelling technique, which are pictorial representations of the physical system through power flow paths and offers flexibility to the modeller besides formulating the system equations.

2. Physical system

The vertical column or spinal column is called backbone, which is curved bony rod present on dorsal side of neck and trunk. This part is having four curves named as cervical and lumber curves directed forward, while the sacral and thoracic curves are directed backward. Out of these curves, lumber curves especially help in erect posture of body and bipedal locomotion of man. The vertebral column of the man is shown in Fig.1. The present study is only focussed for the lumber vertebrae having five vertebrae’s and Thoracic vertebrae T₁₂ and sacral vertebrae formed by the fusion of five sacral vertebrae are the nearby vertebrae considered in the model. The schematic diagram of the lumber vertebrae is shown in Fig. 2.

![Figure 1: Vertebral Column](image1)

![Figure 2: Schematic diagram of lumber vertebra for modelling](image2)
3. Modeling of Rigid Element of Lumber Vertebrae

In this research work, a bondgraph model of rigid elements for lumber vertebrae has been modelled. In this model, rigid element has been modelled as Rayleigh beam element, which account for bending as well as rotary inertia. The muscle strength and intervertebral fluid have been modelled as spring-damper system. The flexural rigidity of the model also depends on the compression of lumber vertebrae. In this work, T_{12}-L_{1}-L_{2}-L_{3}-L_{4}-L_{5}-S_{1}, these seven elements have been modelled. The overall nonlinear stiffness of the various segments is also considered in the model. Besides this, different postures of the spine with excitation base have been considered for modelling. The strength of the muscle and intervertebral fluid has been modelled considering six spring mass damper system between the rigid elements.

The rigid element model of lumber vertebrae is based on the Rayleigh beam model, where rotary inertia of the beam is included. Rayleigh beam model is a superior model than the Euler-Bernoulli model due to inclusion of the rotary inertia of the beam in the model, though shear deformation is neglected and it will be incorporated in near future by assuming the beam element as Timoshenko beam element. The stiffness of the rigid elements relates the generalized Newtonian forces to the generalized displacements at the ends of the element as given by the following equation in XZ direction:

\[
\begin{bmatrix}
V_{xl} \\
M_{xl} \\
V_{xr} \\
M_{xr}
\end{bmatrix}
= [K] \begin{bmatrix}
x_l \\
\psi_l \\
x_r \\
\psi_r
\end{bmatrix},
\]  

(1)

In Eq. (1), \( V_{xl} \) and \( V_{xr} \) are the generalized forces, \( M_{xl} \) and \( M_{xr} \) are the bending moments whereas \( x_l, x_r, \psi_l \) and \( \psi_r \) are the generalized displacements. Subscript “l” and “r” represents the upper and lower side of the model in XZ direction. The stiffness matrix of the rigid element can be modeled as a 4-port C-field storing energy due to the four generalized displacements as shown in Fig.3. Any column of the stiffness matrix may be determined by assigning unit value to the corresponding row element of the displacement vector putting all other elements zero. The equilibrium forces and moments are evaluated on an element satisfying the following equation to the elastic line

\[
M(x) = EI \frac{d^2}{dx^2} y(x).
\]  

(2)

In terms of flexural rigidity \( EI \) and element length \( l \), the stiffness matrix of the vertebrae column may be given as

\[
[K] = \frac{EI}{l^3} \begin{bmatrix}
12 & 6l & -12 & 6l \\
6l & 4l^2 & -6l & 2l^2 \\
-12 & -6l & 12 & -6l \\
6l & 2l^2 & -6l & 4l^2
\end{bmatrix},
\]  

(3)

and finally compliance field in XZ direction may be represented by

\[
\begin{bmatrix}
V_{xl} \\
M_{xl} \\
V_{xr} \\
M_{xr}
\end{bmatrix}
= \frac{EI}{l^3} \begin{bmatrix}
12 & 6l & -12 & 6l \\
6l & 4l^2 & -6l & 2l^2 \\
-12 & -6l & 12 & -6l \\
6l & 2l^2 & -6l & 4l^2
\end{bmatrix} \begin{bmatrix}
x_l \\
\psi_l \\
x_r \\
\psi_r
\end{bmatrix}.
\]  

(4)

Compliance field in YZ direction may also be represented as
\[
\begin{bmatrix}
V_{yl} \\
M_{yl} \\
V_{yr} \\
M_{yr}
\end{bmatrix} = \begin{bmatrix}
12 & 6l & -12 & 6l \\
6l & 4l^2 & -6l & 2l^2 \\
12 & 2l^2 & 6l & 4l^2 \\
6l & 4l^2 & -6l & 2l^2
\end{bmatrix}\begin{bmatrix}
y_l \\
\theta_l \\
y_r \\
\theta_r
\end{bmatrix}.
\] (5)

In Eq. (5), \( V_{yl} \) and \( V_{yr} \) are the generalized forces, \( M_{yl} \) and \( M_{yr} \) are the bending moments whereas \( y_l, y_r, \theta_l \) and \( \theta_r \) are the generalized displacements in YZ direction. A bond graph model of the beam element may now be created by lumping the element inertias and appending them to 1-junction representing displacement and rotation at the ends of the elements. The stiffness of the vertebrae may be modeled as 4-port compliance fields storing energy due to the four generalized displacement as shown in Fig. 3. The two compliance fields model the bending of the vertebrae in XZ and YZ directions respectively. Internal damping of the bones (\( R_{ex} \)) is also considered in the model in the fixed X and Y frame in the model. Masses and rotary inertia have been lumped as its top and bottom ends.

4. Modelling of Intervertebral Fluid

The muscle strength and intervertebral fluid is modelled considering equivalent to mechanical spring-damper system, which is shown in Fig.4a and 4b.

The spring and damper in general reacts to the relative velocity across them. To properly add the velocity component at each end of these elements, the spring and damper are assumed to be positive in compression.
5. Conclusions

The bond graph model of the lumber vertebrae will be simulated using a Runga-Kutta fourth order method on software SYMBOLS-Shakti (Mukherjee, 2005) and some interesting phenomena of the vertical acceleration and deflection of the lower spine in different postures may be evaluated after the simulation.

It has been concluded in the paper that the bondgraph representation of lumber vertebrae represents the accurate representation of flexural dynamic behaviour of the spinal system.

Reference