Mathematical Modeling on Diabetes Mellitus under the Administration of Quantitative Diet using Joslin’s principle for various Body Frames

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Abstract

Mathematical model is presented using the administration of palatable composition of quantitative diet and insulin to study hyperglycemia and hypoglycemia. Blood sugar and insulin levels are calculated under the palatable composition of protein (P), fat (F) and carbohydrate(C). Men and Women aged 25 years and above and juvenile aged ½ year and upto 20 years with three different body frames. Fasting blood glucose 200 mg /100 ml or more and fasting blood glucose above 140mg/100ml with a high post prandial blood glucose are the conclusive values of hyperglycemia to correlate the non-palatable diet among the various body frames. In the present model, the inputs of quantitative diets are chosen as 700 to 2700 calories per day for Juvenile (aged ½ year to 20 years), 1300 to 2700 calories per Men and Women (aged 25 years and above) with three different body frames. Closed form solutions are obtained for solving the simultaneous differential equations for blood sugar and insulin levels using Joslin’s principle for palatable composition of quantitative diet.

Key words: diabetes mellitus, hyperglycemia and palatable diet.

1. Introduction

Diabetes is the disorder of metabolism causing excessive thirst and the production of large volume of urine containing excess of sugar. Metabolic disorder is of two types. Diabetes insipidues: this is the rare metabolic disorder in which the body passes large quantity of colorless urine that contains more water causing thirsty, dry hands, constipation.

Diabetes mellitus is the disorder of carbohydrate metabolism in which sugars in the body are not oxidized to produce energy due to lack of pancreatic hormone insulin. The accumulation of sugar leads to cause of hyperglycemia in the blood.

The disorder may be triggered by the various factors including physical stress, weight less, retinopathy and hardening of the arteries (leads to atherosclerosis). Also due to unoxidised glucose, the cells in the lining of blood vessels and other organs get damaged. Among three types of islets cells α, β and δ, the β cells secrete 70% - 80% insulin. Due to injury or other regions of islets cells, β cells fails to secrete insulin and as a result underutilization of glucose causes polyuria, glycosuria, weight loss, weakness, retinopathy and neuropathy due to thickening of arteries. In order to regulate the proper utilization of blood glucose, administration of insulin and palatable diet distribution are necessary for varying body frames to maintain the proper utilization of insulin for the metabolic activities. Palatable diet under equally divided meals will help the usage of reasonable doses.

A general approach of Gatewood et al. [1970] explains the effect of hormone glucose and insulin secreted by the pituitary and thyroxin produced by thyroid as simplest mathematical form. Katiyar et al. [2003] discussed the regulation of blood glucose level
in diabetes mellitus using palatable diet composition. Villegas et al. [2007] analyzed that high intake of foods with a high glycemic index and load, especially rice, the main carbohydrate contributing food may increase the risk of type-II diabetes mellitus in Chinese women. We have presented the mathematical model for the study of palatable composition of quantitative diet in diabetic mellitus for various body frames.

2. Formulation

Model refers to the quantities: x for blood sugar level, y for blood insulin level, z for food input and I for insulin input. Under the assumptions of the normal fasting level of blood glucose with 70-100 mg/100 ml before breakfast and 120-140 mg/100 ml following a meal, the gradients of blood sugar (x) and the insulin (y) can be modeled as,

\[
\frac{dx}{dt} = -\alpha x y + \beta x_0 H(x_0 - x) - \beta x H(x_0 - x) + \gamma Z(t) \\
\frac{dy}{dt} = (\phi x)H(x - x_0) - (\phi x_0) H(x - x_0) - \psi y_0 + \xi I(t)
\]

Where \(\alpha\), \(\beta\), and \(\gamma\) are the positive constants called sensitivity values for insulin, the low blood sugar level, high blood sugar level and the input level respectively for the sugar level gradient, \(\phi\), \(\psi\) and \(\xi\) are the positive constants and are called the sensitivities for high blood sugar level, insulin level and the input level respectively, ‘H’ is the unit step function.

The input to the blood sugar level is via food source. The food store is filled periodically and the contents at any stage are reduced in a simple exponential manner. Therefore the source term \(Z(t)\) can be written as,

\[
Z(t) = \begin{cases} 
0 & , \quad t < t_0 \\
Q e^{-\delta(t-t_0)} & , \quad t \geq t_0 
\end{cases}
\]

Where \(Q\)-quantity of meal, \(\delta\)- delay parameter, \(t_0\) - time of the meal. The subcutaneous injection at periodic intervals leaks its contents into the system over a period of time. Then we take the maximum effect to mean leakage rate as \(I(t)\) and is modeled as

\[
I(t) = \frac{\rho t_1}{t_1-t_0} + bt + k , \quad \text{Let } t_1-t_0 = \Delta t = 3.5 \text{ hours} \quad \Delta t = \frac{14}{4} = 3.5 \text{hours}
\]

When \(t = 3.5 \text{ hours}\), the food intake \(I(t)\) is given by \(I(t) = \rho + (3.5) b + k\)

\[
z(t) = Q e^{-\delta(t_1-t_0)} \text{ where } t_1-t_0 = \frac{\rho t_1}{I(t) - b t - k} , \text{where } \rho \text{ - quantity of injection}
\]

\(t_0\) - time of injection, \(t_1\) - time of maximum for one meal, \(b\) - is the slope up to the ramp.

3. Analysis

General form of equations (1) and (2) describe the blood sugar and insulin levels. Changing the non linear term \(\alpha xy\) as \(\tilde{\alpha} xy\) (linear term of \(x\) and linear term of \(y\) are to be targeted as independent solutions) then, for \(x > x_0\), \(H(x_0 - x) = 0\)
for \( t \leq t_0 \), \( z(t) = 0, \ I(t) = 0, \) for which equations (1)\&(2) become

\[
\begin{align*}
x_{PD}^{(t)} &= C_1 e^t \sin t + C_2 e^t \cos t + \gamma \frac{Q^P}{(\gamma^2 - \alpha^2)} e^{\frac{-4.16(t - t_0)}{\delta^2 - \gamma^2}} \frac{\mu t}{\beta} + \frac{\psi \mu - \alpha x \lambda \phi}{\alpha x \phi^2} \\
y_{PD}^{(t)} &= (C_1^{PD} + C_2^{PD})(0.25) e^t \sin t + (-C_1^{PD} + C_2^{PD})(0.25) e^t \cos t + \frac{\psi \gamma Q^P e^{-\delta (t - t_0)}}{\delta^2 - \gamma^2} \frac{\mu}{\alpha x \phi} + \frac{\psi \mu - \alpha x \lambda \phi}{\alpha x \phi^2}
\end{align*}
\]

(6)

Since \( t \leq t_0 \) implies \( t_0 \leq t \leq \infty \), for which the time for whole meal cannot be determined absolutely, but we consider half the time of the meal, from equation (3),

\[
\int_{t_0}^{\infty} z(t) \, dt = \int_{t_0}^{\infty} Q e^{-\delta (t - t_0)} \, dt, \quad z(t) = Q e^{-\delta t} \left\{ -\frac{\rho}{I(t) - b} \frac{t_1}{t} \right\}
\]

(8)

For food source \( z(t) \) at 4 different meals at the rate of [1473 – 2255] calories for men aged 25 years and above, [1475 – 2040] calories for women aged 25 years and above and [1505 – 2430] calories for Juvenile aged ½ year to 20 years.

We consider the quantity of the meal (Q) in calories with palatable choice of diet using Joslin’s Principle; we take the ratio of protein (P), fat (F), and carbohydrates(C) as

Defining \( x = x^{PD} \), \( y = y^{PD} \), \( C_1 = C_1^{PD} \), \( C_2 = C_2^{PD} \), \( Q = Q^{PD} \), With \( 'PD' \) = palatable diet.

Analyzing the blood sugar and insulin levels by providing the palatable diet at four different meals divided to four different values of \( t \), it is at \( t = t_0 = 6 \) hours (break fast), \( t_1 = 12.00 \) hours (noon), \( t_2 = 18.00 \) hours (evening) and \( t_3 = 12.00 \) (late evening), the types of palatable diets are computed. Most normal cases show a natural period of less than three hours, whereas most diabetic cases show a natural period of greater than five hours. For the liquid diet we take \( x(t_0) = 524 \) (first feeding), 504 (second feeding), 504 (third feeding) and 495 (fourth feeding) calories correspondingly \( C_1^{PD} \) and \( C_2^{PD} \) are calculated. The constants \( C_1^{PD} \) and \( C_2^{PD} \) have been computed analytically for four different meals in relation to liquid and house diets. The palatable composition is computed to quantify the glucose levels.

4. Methodology

\[
\frac{dx}{dt} = -\alpha x^2 y \quad \text{and} \quad \frac{dy}{dt} = \varphi x - \varphi x_0 - \psi y, \text{with}
\]

\( \alpha = 0.05, \ \beta = 1.0, \ \gamma = 4.0, \ \psi = 2.0, \ \varphi = 80 \) and \( \varphi = 5.0 \)

\( t_0 = 4, \ x_0 = 17.5, \ y_0 = 1 \) and \( h = 4 \)

Then \( x \) and \( y \) are, \( x_1 = x_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4] \ y_1 = y_0 + \frac{1}{6}[l_1 + 2(l_2 + l_3) + l_4] \)

4.1 Models for Q with palatable diet for 14 samples under various body frames

Men & Women aged 25 years and above and Juvenile aged ½ year up to 20 years with weight 58 kg to 83 kg, 52 kg to 71.5 kg and 7 kg to 60 kg respectively
5. Results and discussions

On observing the palatable composition as Protein: Fat: Carbohydrates (P: F: C), it is cleared that glucose is an important metabolic fuel, in general, (the percentage of carbohydrate ratio is more than protein and fat) for the digestion process in relation to the oxidation of P: F: C. Because the input of the carbohydrate will be well utilized by the brain, insufficient quantities to meet the energy demand. The measurements of blood sugar x (t) and insulin y (t) from equations (5) and (6) are functions of time t. The graphs are obtained for 13 pairs for three different categories Juvenile (J), Men (M) and Women (W) for varying body frames. For the sensitivity parameters $\alpha = 0.05$, $\beta = 0.98$ and $\gamma = 3.95$, the glucose tolerance has little effect on blood glucose in response to $\varphi = 0.5$ and $\psi = 1.962$. The response for glucose rises rapidly and falls to zero within two hours, the appearance of $x_0$ and $y_0$ provide two-parameter classification of blood glucose control.

Figures (1) and (2) describe the blood glucose and insulin levels nearly to normal range under palatable composition of diet for men aged 25 years and above,

**Palatable input range of P: F: C** (for four equal meals, for fourteen samples)
- For medium body frame : 50:70:150- 80:100:260gm [1430 - 2260] cal

Figures (1) and (3) describe the blood glucose and insulin levels nearly to normal range under palatable composition of diets. For women aged 25 years and above palatable input range of P: F: C

**Palatable input range of P: F: C** (for four equal meals, for fourteen samples)
- For large body frame : 50:45:100 - 100:110:260gm [1505 - 2430] cal

Figures (1) and (4) describe the blood glucose and insulin levels nearly to normal range under palatable composition of diet for Juvenile cause ½ years to 20 years

**Palatable input range of P: F: C** (for four equal meals, for fourteen samples)
- For average body frame : 40:35:90 - 90:100:247gm [835 - 2248] cal
- For medium body frame : 40:45:105 - 95:105:255gm [1032 - 2345] cal
- For large body frame : 50:45:100 - 100:110:240gm [1505 - 2430] cal

<table>
<thead>
<tr>
<th>Table-1: Palatable diet composition rate per sample (taken for 14 samples, each is averaged for 10 samples)</th>
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<tbody>
<tr>
<td><strong>Body frame/composition</strong></td>
</tr>
<tr>
<td><strong>Protein (P)</strong></td>
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<td></td>
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<td></td>
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<td><strong>Fat (F)</strong></td>
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<tr>
<td><strong>Carbohydrates (C)</strong></td>
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<table>
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<tr>
<th>Fat (F)</th>
<th>1.42857</th>
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<th>1.7857</th>
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<th>1.4285714</th>
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<tr>
<td>Carbohydrate(C)</td>
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<td>7.14285</td>
<td>6.7857</td>
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<td>38.71429</td>
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### References